Towards Stable Adaptive Laws in Qualitative Model Reference Adaptive Control

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Abstract

In this paper a method to derive stable adaptive laws for a Model Reference Adaptive Control system based on qualitative (less precise) models is presented. Qualitative, lower order models represent the underlying physical system imprecise but accurate. Further, multi-valued performance specifications can be expressed explicitly by a qualitative model in the form of a reference model. The Qualitative Model Reference Adaptive Control system utilises a fuzzy qualitative simulation method to represent this reference model and the fuzzy dynamical process model. These fuzzy dynamical models are of an analytic form and allow system analysis to be based on interval analysis. Therefore, stability of the adaptive loop in Qualitative Model Reference Adaptive Control is based on results for the stability of interval systems. In this paper a method towards stable adaptive laws based on results from robust parametric systems is presented.

1 Introduction

For many application domains control systems based on simple linear, time-invariant models have been spectacularly successful. However, for some domains the assumption of linearity and time-invariance are often invalid under practical constraints. In these cases either higher order models, non-linear models or adaptive systems are used to cope with the complex practical environment. These approaches need analytic complexity and performance, e.g. stability, can only be assured under restrictive conditions.

This paper proposes an alternative approach to cope with complexity by utilising less precise, i.e. fuzzy qualitative, models that can maintain accuracy over an extended range of operation. This allows low order less precise models to be used to generate the desired behaviour. This paper provides a stability analysis of a Model Reference Adaptive Control (MRAC) system based on fuzzy qualitative simulation.

Fuzzy qualitative simulation methods simulate fuzzy dynamical models to obtain knowledge about the behaviour of the underlying system. Fuzzy dynamical models are a set of Fuzzy Differential Equations (FDEs). These differential equations are derived from theoretical knowledge about the physical system. The parameters of the FDEs are fuzzy sets. Fuzzy sets incorporate subjective knowledge about the physical system to build less precise models based on subjective and theoretical knowledge. FRenSi, a fuzzy qualitative simulator, is used in this approach to simulate the fuzzy reference model in the Qualitative Model Reference Adaptive Control (QMRAC) system [1].

Control specifications are often multi-valued due to a trade off between conflicting performance criteria, e.g purity versus energy costs in a distillation process. MRAC has the advantage of a reference model to express the desired control performance explicitly. In most of the MRAC applications the desired performance is given by a numerical linear model. This model cannot reflect imprecise performance specifications. Further, the primary control loop due to its approximate process model cannot be expected to follow this unique reference behaviour perfectly. A less precise reference model in the form of a fuzzy dynamical model incorporates multi-valued performance specifications explicitly.

Model Reference Adaptive systems modify the control system such that the actual behaviour follows or tracks the desired behaviour generated by the reference model. Numerical MRAC approaches have developed design methods for the adaptive updating of the control system. These normally are based on methods that support a stability analysis, generally time-varying and non-linear. QMRAC can be seen as a variable precision controller because the performance specification can be easily varied in its precision. The theoretical background of FDEs allows to deal with fuzzy dynamical models in an analytic way. Therefore, the system design is kept as analytic as possible, e.g. analytic primary controllers are preferred to heuristic controllers.

Stability analysis plays an important role in design and development of control systems and in particular MRAC. With the analytic property of the fuzzy dynamical models a stability proof for QMRAC can be derived. This proof is based on the results by Kharitonov [2] for interval polynomials and results by Chapellat [3]. In [3] Kharitonov's theorem is extended to nonlinear interval control systems, i.e the interval Lur'e problem. Stability in QMRAC is based on these results.

2 Fuzzy Qualitative Simulation

The fuzzy qualitative simulation method used in this approach is important for the design and the analysis of the system. FRenSi (Fuzzy Region Simulator) is employed to simulate the less precise, fuzzy dynamical models. FRenSi is a numerical simulator that can solve FDEs. The process and reference model in QMRAC are fuzzy dynamical models which are a set of FDEs.

The simulator combines the fuzzy sets from the model parameters and the initial conditions to form a hypercube, called fuzzy region. The fuzzy region is sampled explicitly and numerical simulations generate the evolution of this fuzzy region. The fuzzy region is maintained during the entire simulation to keep the interaction between the system variables. Interaction is the mutual constraints between system variables introduced by previous integration steps. Mapping back to individual fuzzy sets with a following simulation step based on these new fuzzy sets would loose this information. In FRenSi the compilation of individual fuzzy sets is for external purpose only, i.e. output behaviour of the model. Any subsequent integration step proceeds on the fuzzy region from the previous step. Due to its interacting method the simulator does not introduce spurious solutions, which most other qualitative or interval simulators do. Spurious solutions are simulated behaviours that the underlying physical system cannot reach. The simulator produces the smallest possible description of the future system state based on a fuzzy dynamical model. The state description is a range on the real number line varying in its precision and uncertainty represented by time-varying fuzzy sets. In the following example the fuzzy state of second order fuzzy dynamical models is calculated.

Given a fuzzy dynamical model in form of a second order Fuzzy Differential Equation

$$\ddot{\overline{X}}(t) = -\overline{A}_1\dot{\overline{X}}(t) - \overline{A}_2\overline{X}(t) + \overline{B}_0U(t)$$

with the parameter and initial conditions as triangular fuzzy sets, $\overline{A}_1 = (0.54, 0.6, 0.66)$, $\overline{A}_2 = (0.95, 1, 1.05)$, $\overline{B}_0 = (1, 1, 1)$, $\overline{X}(0) = (0, 0, 0.1)$ and $\dot{\overline{X}}(0) = \ddot{\overline{X}}(t) = (0, 0, 0)$ the simulation is shown in Figure 1. The figure

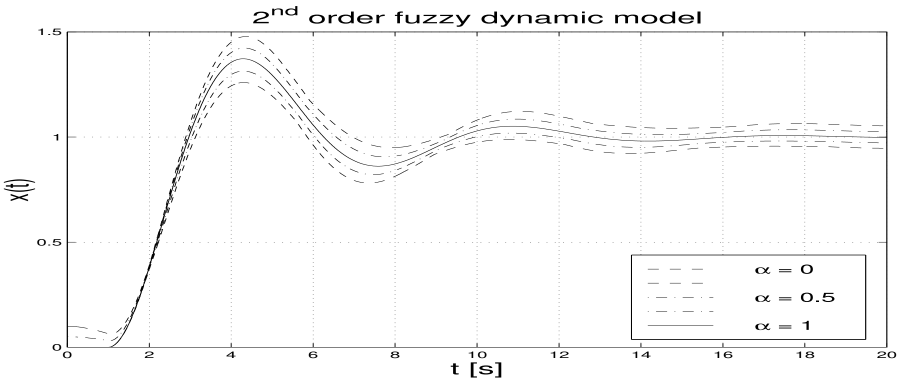

Figure 1: FRenSi simulation

shows the response of the fuzzy dynamical model with an input step from 0 to 1 at $t = 1s$. The initial state $x(0)$ is imprecise and the parameters are fuzzy. This leads to a bunch of system trajectories represented by the borders of the α-cuts.

The extension principle of Zadeh and Nguyen's identity are the theoretical basis of this simulation approach. These two lemmas build the analytical background for the analysis with fuzzy dynamical models.

3 Qualitative Model Reference Adaptive Control (QMRAC)

The QMRAC system is designed and developed on the basis of imprecise and uncertain process and specification knowledge in form of fuzzy dynamical models. The design uses these explicit models in an analytic way to develop an adaptive controller. The possibility of variable precision in the models and therefore in the controller makes the QMRAC system an extension to the MRAC approach. The structure of QMRAC, shown in Figure 2, is similar to a parallel MRAC system. The primary controller is preferably any analytic controller, e.g. PID

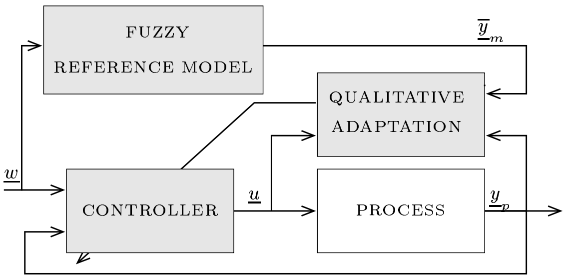

Figure 2: Qualitative Model Reference Adaptive Control System

or Fuzzy Equation-Based Controller. The analytic structure of controller and process model helps to calculate the change in the control signal when adapting the controller parameters. Further, analytic controllers allow analytic design and a stability analysis of the system.

In QMRAC the multi-valued performance specification is made explicit by the fuzzy reference model. For example, the imprecise performance specifications are mapped onto fuzzy parameters of a second order fuzzy dynamical model. This specifies a range of time constants and gain in the time domain or a range of damping factor, frequency and gain in the frequency domain. The desired range of the system performance can be evaluated by FRenSi simulations. Taking into account the fuzzy dynamical process model and the chosen controller structure the reference model is developed in an recursive procedure to ensure the system can reach the desired performance.

The adaptation has to find suitable changes of the controller parameters to ensure the desired performance of the primary loop. The reference model is fuzzy and therefore provides a fuzzy reference behaviour as output. First, a suitable tracking error between this fuzzy reference behaviour and the process measurements is generated. Secondly, an adaptive filter is designed on the basis of fuzzy dynamical models with a fuzzy tracking error as input.

The fuzzy tracking error is generated by a distance measurement based on the Generalized Hausdorff Distance by Dubois and Prade [4], which has been altered to generate a direction sensitive distance measure. This is important in order to change the controller parameters in the correct direction.

The adaptive laws are derived from their real-valued counterparts, e.g. gradient method [5] or hyperstability. The real-valued adaptive law based on fuzzy dynamical models is extended to fuzzy sets under the extension principle.

3.1 Stability Analysis

In the QMRAC approach fuzzy dynamical models are utilised whose analytic form is used to determine the stability of the system. The advantage of fuzzy sets in regard to stability analysis is that fuzzy sets can have a finite range on the real number line. Many fuzzy sets, e.g. triangular and trapezoidal sets, do have this feature. This makes stability analysis possible as there exists a definite and limited range of behaviours. In stochastic control approximations have to be made to represent the probability distribution by statistical properties therefore stability cannot be guaranteed. Kharitonov's work [2] on stability of a family of linear differential equations builds the foundation for the QMRAC stability proof. A linear fuzzy dynamical model is a family of linear differential equations with different level of confidence. For system stability only worst case calculations are of interest which leads to the family with the largest imprecision, i.e. the lowest α-cut ($\alpha = 0$).

Kharitonov's theorem is directed towards the asymptotic stability of a family of ordinary differential equations. Sufficient conditions are provided under which the zeros of the characteristic interval polynomials have all negative real parts and therefore all the systems of this family are stable. To ensure stability of the family of these linear systems, only four polynomials have to be checked.

Theorem 1 (Kharitonov) *The family of polynomials*

$$P(s, \mathbf{A}) = \{p(s, \mathbf{a}) = a_0 + a_1 s + \ldots + a_n s^n \mid \mathbf{a} \in \mathbf{A}\}, a_n > 0$$

is stable if the following four polynomials

$$\begin{aligned}
K_1(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \ldots \\
K_2(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \ldots \\
K_3(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \ldots \\
K_4(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \ldots
\end{aligned}$$

are stable.

These four polynomials are referred to as Kharitonov polynomials. The superscript $^-$ and $^+$ indicates the lower and upper limit of the coefficients.

The result from Kharitonov has initiated renewed interest in the field of robust parametric system, especially in the field of control. In [3] the stability of a linear time-invariant interval system with a non-linear feedback block is investigated. Conditions for passivity and strictly positive realness of interval plants are presented. Given a family $\mathcal{G}$ of interval plants with

$$g(s) = \frac{n(s)}{d(s)}$$

where $n(s)$ belongs to a family of interval polynomials $\mathcal{N}$ and $d(s)$ belongs to family of interval polynomials $\mathcal{D}$ defined as follows:

$$\begin{aligned}
\mathcal{N} &= \{n(s) : n(s) = n_0 + n_1 s + \ldots + n_m s^m \ \wedge \ n_i \in [n_i^-, n_i^+], \ \forall i = 0, \ldots, m\} \\
\mathcal{D} &= \{d(s) : d(s) = d_0 + d_1 s + \ldots + d_n s^n \ \wedge \ d_i \in [d_i^-, d_i^+], \ \forall i = 0, \ldots, n\} .
\end{aligned}$$

Following theorem gives the sufficient condition for a family of interval plants to be strictly positive real (SPR):

Theorem 2 *Every plant $g(s)$ in the family $\mathcal{G}$ is SPR, iff it is the case for the eight following plants:*

$$
\begin{array}{ll}
g_1(s) = \dfrac{n^{++}(s)}{d^{+-}(s)} & g_2(s) = \dfrac{n^{-+}(s)}{d^{+-}(s)} \\
g_3(s) = \dfrac{n^{+-}(s)}{d^{++}(s)} & g_4(s) = \dfrac{n^{--}(s)}{d^{++}(s)} \\
g_5(s) = \dfrac{n^{+-}(s)}{d^{-+}(s)} & g_6(s) = \dfrac{n^{--}(s)}{d^{-+}(s)} \\
g_7(s) = \dfrac{n^{++}(s)}{d^{--}(s)} & g_8(s) = \dfrac{n^{-+}(s)}{d^{--}(s)} ,
\end{array}
$$

with $n^{++}(s) = n_0^+ + n_1^+ s + n_2^- s^2 + n_3^- s^3 + \ldots$ This theorem states that only 8 of the 16 Kharitonov systems $\mathcal{G}^K$ have to be checked to ensure the strictly positive realness. Further, this ensures that the interval family $\mathcal{G}$ is stable and its Nyquist plot is completely in the right half s-plane.

Assuming that the non-linearity f in the feedback path is continuous and belongs to the class $\mathcal{F}_k$ defined by

$$
\mathcal{F}_k = \begin{cases} f(0) = 0 \\ 0 \le yf(y) \le ky^2 \end{cases}
$$

such a non-linearity is said to belong to the k-sector, $[0, k]$.

The well-known absolute stability result for a stable linear system $g(s)$ with a bounded non-linearity f in the feedback path is stated as follows

Theorem 3 *If g(s) is stable and f belongs to the k-sector, it is sufficient for absolute stability that*

$$
1/k + Re\{g(j\omega)\} > 0 \quad \forall \omega \ge 0 .
$$

In any real system the linear feedforward part will be subject to perturbations as well. In the case of parametric perturbations, these can be modelled by an interval system $\mathcal{G}$. Then, the 16 Kharitonov systems $\mathcal{G}^K$ determine the size of a sector for which the non-linear system $\mathcal{G}^f$ is absolutely stable. This is stated in the following theorem.

Theorem 4 *The non-linear system $\mathcal{G}^f$ is absolutely stable if the the interval system $\mathcal{G}$ is stable and the non-linearity f belongs to the sector [0,k] such that*

$$
k < \infty \qquad \textit{if} \quad \inf_{\mathcal{G}^K} \inf_{\omega \in \mathbb{R}} Re\{g(j\omega)\} \ge 0
$$

otherwise

$$
k < \frac{1}{\inf_{\mathcal{G}^K} \inf_{\omega \in \mathbb{R}} Re\{g(j\omega)\}} .
$$

To ensure stability of the linear part only the 8 Kharitonov system from Theorem 2 have to be checked but for generating the minimum value of the real part of the family of transfer functions all 16 Kharitonov system $\mathcal{G}^K$ have to be calculated.

The QMRAC system can be transfered to an equivalent system with a linear time-invariant feedforward block and a non-linear time-varying feedback block. The linear part contains the fuzzy reference model and provides the tracking error as output. The feedback block contains the adaptive law and is non-linear and time-varying. Applying Theorem 2, the strictly positive realness of the feedforward block can be checked. In some cases a linear filter has to be introduced to ensure the SPR property of the feedforward block. The chosen adaptive law must be of the form to ensure that the non-linear feedback block is within the sector $[0, k]$, where k is calculated by Theorem 4. If the non-linear feedback block lies within this sector the QMRAC system is proved to be stable. In the other case the adaptive law has to be altered to ensure the non-linearity is within the required sector. In this way results from robust parametric control can be used to ensure stability of our QMRAC system.

4 Simulation Results

In the following, simulations from a QMRAC system are presented. Figure 3 shows the adaptation for a partially unknown and time-varying process modelled by a second order fuzzy dynamical model. The multi-valued performance specification is mapped onto the gain, damping-factor and frequency of a second order fuzzy reference model. The chosen primary controller is a state feedback controller with. The derivatives for

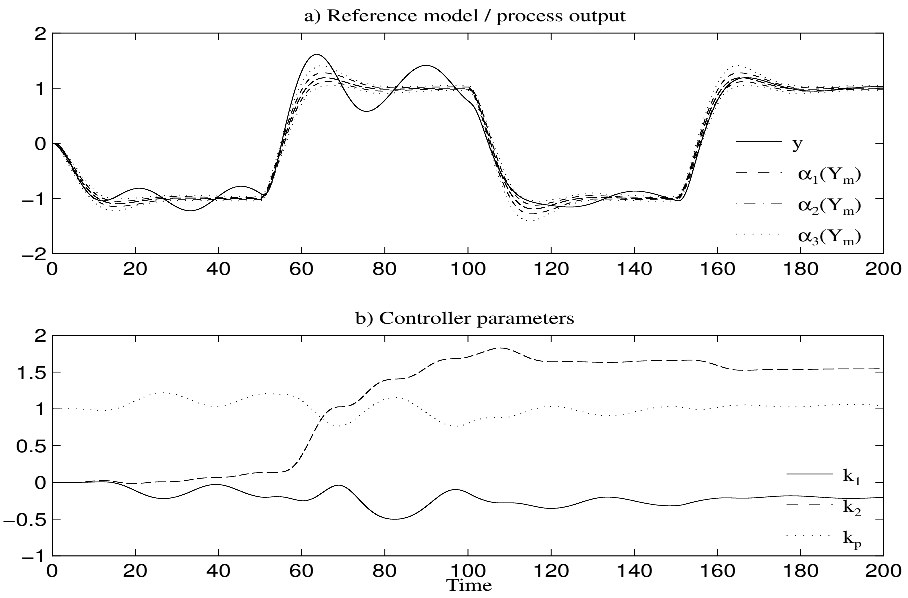

Figure 3: QMRAC simulation

the controller are generated by state variable filters from the process output. The simulation shows that the QMRAC controller succeeds in adapting the process to the desired range of performance. The higher order process which is model by a low order imprecise model, causes an initial displacement between primary loop and desired performance. The plots show that the process output, solid line, finally matches the desired range of performance and the controller parameters reach a local optimum. It is demonstrated that the chosen stable adaptive law is able to adapt the primary loop to achieve a tracking error of zero. The stability of the qualitative adaptive controller can be checked be applying the results from robust parametric control represented above.

5 Conclusion

In this paper a MRAC system is presented which is based on qualitative, less precise, models. These models incorporate theoretical and subjective knowledge from the underlying physical system. Further, multi-valued performance specifications are expressed explicitly through a fuzzy reference model. Stability issues in QMRAC are discussed on the basis of results from interval systems, i.e. robust parametric systems. Especially, the results from [3] enable to assess stability of the QMRAC system. Further work has to be done to get a complete framework for developing stable adaptive laws based on fuzzy dynamical models.

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