Classification of signals

- Deterministic
  - Periodic
    * Sinusoid
    * Non-sinusoid
  - Non-periodic
    * Quasi-periodic
    * Transient
- Stochastic
  - Stationary
    * Ergodic
    * Non-ergodic
  - Non-stationary

Description of signals

Periodic signals

$$x(t) = x(t+T) \tag{1}$$

$$c_n = \frac{1}{T}\int_0^T x(t)\, e^{-j2\pi n\frac{t}{T}}\, dt \tag{2}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{j2\pi n\frac{t}{T}}, \tag{3}$$

Transient signals

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt \tag{4}$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt \tag{5}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df \tag{6}$$

Discrete transient signals

$$E = \sum_{n=0}^{N-1} x^2(n) \tag{7}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi\frac{nk}{N}} \tag{8}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi\frac{nk}{N}} \tag{9}$$

Stochastic signals

$$R_x(\tau) = E\{x(t)\, x(t+\tau)\} \tag{10}$$

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau} d\tau \tag{11}$$

Ergodic stochastic signals

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau)\, dt \tag{12}$$

Discrete ergodic signals

$$R_x(k) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, x(n+k) \tag{13}$$

$$S_x(f) = \sum_{n=-\infty}^{\infty} R_x(n)\, e^{-j2\pi nf} \tag{14}$$

Linear systems

Relation between the input (x) and output (y)

In the time domain:

$$y(t) = \int_{-\infty}^{\infty} x(t-u)\, h(u)\, du \tag{15}$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \tag{16}$$

In the lag domain:

$$R_{x,y}(\tau) = \int_{-\infty}^{\infty} R_x(\tau-u)\, h(u)\, du \tag{17}$$

$$R_y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_x(\tau-u+v)\, h(u)\, h(v)\, du\, dv \tag{18}$$

$$R_{x,y}(n) = \sum_{k=-\infty}^{\infty} R_x(n-k)\, h(k) \tag{19}$$

$$R_y(n) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} R_x(n-k+l)\, h(n)\, h(l) \tag{20}$$

In the frequency domain:

$$Y = HX \tag{21}$$

$$S_y = |H|^2 S_x \tag{22}$$

Computing methods

DFT and FFT

- DFT:
$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi\frac{nk}{N}} \tag{23}$$

- FFT: different algorithm, the same result

- DFT: number of instructions proportional with N^2

- FFT: number of instructions proportional with $Nlog_2N$, supported by special instructions and architecture of DSPs

Correlation function

- Direct method:

$$R_{x,y}(k) = \frac{1}{N-k}\sum_{n=0}^{N-1} x(n)y(n+k) \qquad k = (1-N)..(N-1) \tag{24}$$

- Indirect method:

$$\hat{R}_{x,y}(l) = \frac{1}{N-k}\mathsf{IFFT}(X^*Y) \tag{25}$$

where X^* is the complex conjugate of X.

$\hat{R}_{x,y}$ is biased estimator (circular correlation):

$$\hat{R}_{x,y}(k) = \frac{1}{N-k}\sum_{n=0}^{N-1} x(n)y[(n+k)|N] \qquad k = 0..(N-1) \tag{26}$$

where $m|N$ is the m modulo N.

- Unbiased indirect method: To avoid the circularity, the second half of x and y must be filled by zeros.

Convolution

- Tipical application if FIR digital filters
- Direct method supported by special instructions and architecture of DSPs

Implementation of digital filters

FIR filters

$$y(n) = \sum_{k=0}^{N-1} h(k)\, x(n-k) \tag{27}$$

where x is the input and y is the output of the filter.

In frequency domain:

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n} \tag{28}$$

- Direct method not sensitive for rounding
- Always stable
- Supported by special instructions and architecture of DSPs

IIR filters

$$y(n) = \sum_{m=0}^{M} b(m)\, x(n-m) - \sum_{m=1}^{N} a(m)\, y(n-m) \tag{29}$$

In frequency domain:

$$H(z) = \frac{\sum_{m=0}^{M} b(m)\, z^{-m}}{1 + \sum_{m=1}^{N} a(m)\, z^{-m}} = \frac{B(z)}{A(z)} \tag{30}$$

- Direct method according to (29)
- Cascade structure implemented with series of second order direct structures

$$H(z) = K \prod_{k=1}^{[N/2]} \frac{1 + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}} \tag{31}$$

where [N/2] is the smallest integer $\geq N/2$.

- Resonator-based structure

$$\vec{q}(n+1) = [\mathsf{diag}(\vec{z}) - \vec{g}\vec{c}^T]\vec{q}(n) + \vec{g}\,x(n) \qquad (32)$$

$$y(n) = (\vec{w}^T - \vec{c}^T d)\vec{q}(n) + d\,x(n) \qquad (33)$$

where x is the input, y is the output and $\vec{q}$ is the state vector, $\vec{z}$, $\vec{g}$, $\vec{w}$ are the parameter vectors of the filter, $\vec{c}^T = [1, 1, ..., 1]$.

The transfer function of the filter is the following:

$$H(z) = \frac{d + \sum_{n=0}^{N-1} w_n\, H_n(z)}{1 + \sum_{n=0}^{N-1} H_n(z)} \qquad (34)$$

where

$$H_n(z) = \frac{g_n\, z^{-1}}{1 - z_n z^{-1}} \qquad (35)$$