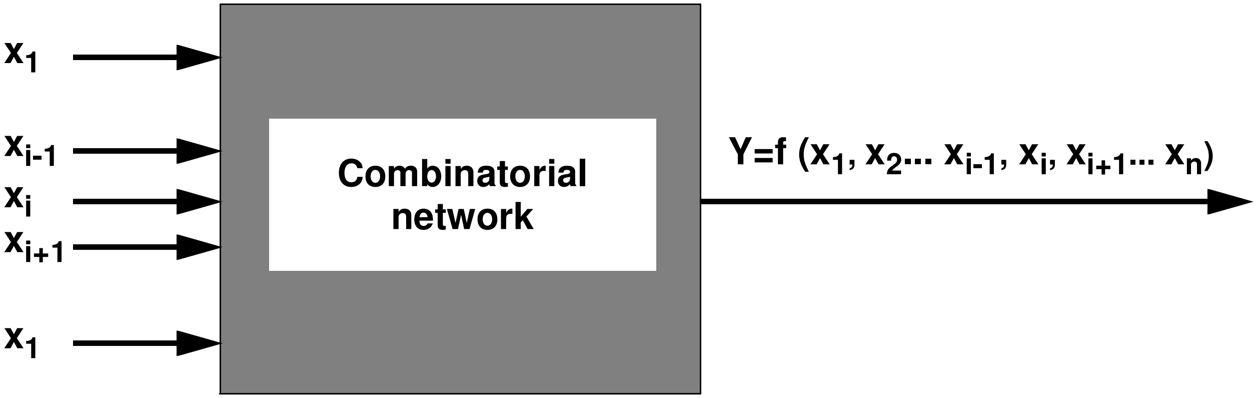
Automatic test generation (ATPG)

Boole differences



- **Lemma (Shannon normal form): Let** $f_{0,i}(\underline{x}) = f(x_1 \ldots x_{i-1}, 0, x_{i+1} \ldots, x_n)$ **and** $f_{1,i}(\underline{x}) = f(x_1 \ldots x_{i-1}, 1, x_{i+1} \ldots, x_n)$ **, then**
  $f(\underline{x}) = f(x_1 \ldots x_{i-1}, x_i, x_{i+1} \ldots, x_n) = x_i \cdot f(x_1 \ldots x_{i-1}, 1, x_{i+1} \ldots, x_n) + \overline{x_i} \cdot f(x_1 \ldots x_{i-1}, 0, x_{i+1} \ldots, x_n)$
  $= x_i \cdot f_{1,i}(\underline{x}) + x_i \cdot f_{0,i}(\underline{x})$
- **s-a-0 (s-a-1) fault on input i ⇒ faulty function:** $f_{faulty}(\underline{x}) = f_{0,i}(\underline{x})$ **(** $f_{1,i}(\underline{x})$ **).**
- **fault detection ⇔** $f(\underline{x}) \neq f_{faulty}(\underline{x})$ **(single output circuit:** $f(\underline{x}) \oplus f_{faulty}(\underline{x}) = 1$ **)**
  - **s-a-0 on input i:**

    $$f(\underline{x}) \oplus f_{\mathbf{faulty}}(\underline{x}) = f(\underline{x}) \oplus f_{0,i}(\underline{x}) = [x_i \cdot f_{1,i}(\underline{x}) + \overline{x_i} \cdot f_{0,i}(\underline{x})] \oplus f_{0,i}(\underline{x}) = x_i \cdot [f_{0,i}(\underline{x}) \oplus f_{1,i}(\underline{x})]$$

  - x_i ***Injection at the*** **fault location (fault sensitization)**
  - $\frac{a}{dx_i} f(\underline{x}) = f_{0,i}(\underline{x}) \oplus f_{1,i}(\underline{x})$ **is the Boolean difference of** $f(\underline{x})$ **by** x_i **(error propagation)**

- ***All test patterns as result based on a functional description***

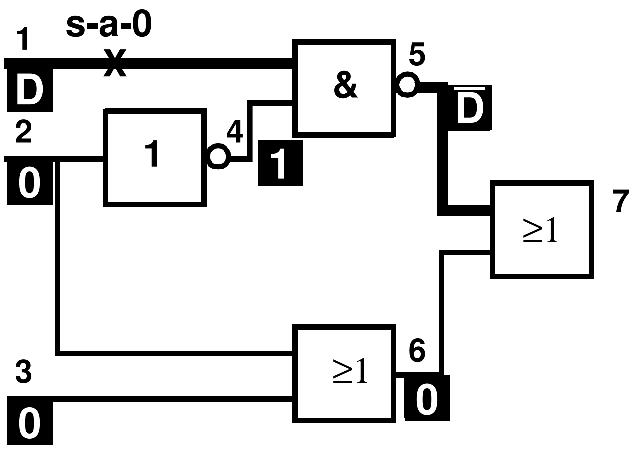
Simultaneous description of the signal values

- **Composite logic values: (value pairs)** ***joint calculation*** **in fault free and faulty circuits**
- **Five valued logic: (fault free/ faulty)**
  0 = 0/0
  1 = 1/1
  x = x/x
  D= 1/0
  $\overline{D}$ = 0/1
- **Boolean Algebra**

&	0	1	x	D	$\overline{D}$
0	0	0	0	0	0
1	0	1	x	D	$\overline{D}$
x	0	x	x	x	x
D	0	D	x	D	0
$\overline{D}$	0	$\overline{D}$	x	0	$\overline{D}$

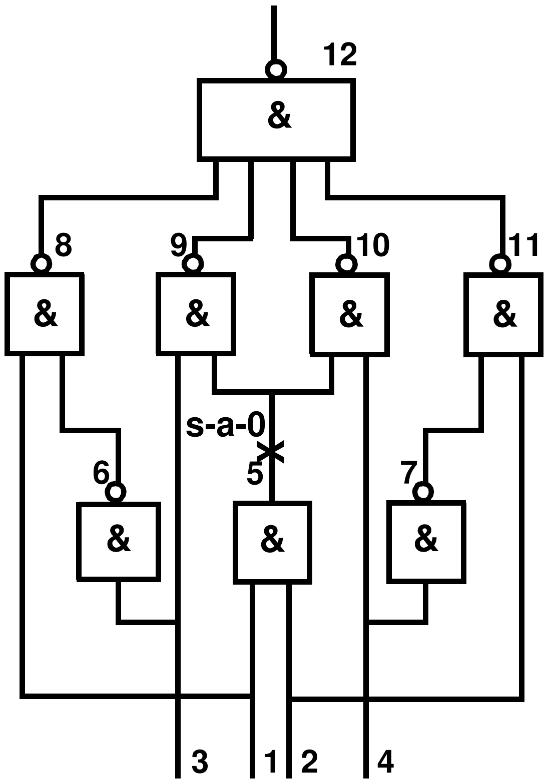
Sensitized path

- **Critical path**



Operation	Node						
	1	2	3	4	5	6	7
Fault injection	D						
Sensitization 1 $\rightarrow$ 5	D			1	$\overline{D}$		
Implication 4 $\rightarrow 2$	D	0		1	$\overline{D}$		
Sensitization 5 $\rightarrow$ 7	D	0		1	$\overline{D}$	0	$\overline{D}$
Implication 2,6 $\rightarrow 3$	D	0	0	1	$\overline{D}$	0	$\overline{D}$

The counterexample of Schneider



Operation	Node											
	1	2	3	4	5	6	7	8	9	10	11	12
Fault injection					D							
Sensitization G. 5	1	1			D							
Propagation 5→9	1	1	1		D				$\overline{D}$			
Implication 3 → 6	1	1	1		D	0			$\overline{D}$			
Implication 6 → 8	1	1	1		D	0		1	$\overline{D}$			
Propagation 9→12	1	1	1		D	0		1	$\overline{D}$	1	1	D
Implication 2,11 → 7	1	1	1		D	0	0	1	$\overline{D}$	1	1	D
Implication 7→4	1	1	1	1	D	0	0	1	$\overline{D}$	1	1	D
Contradiction	1	1	1			0	0	1	$\overline{D}$		1	D
Backtrace	1	1	1		D	0		1	$\overline{D}$			
Backtrace	1	1			D							
Propagation 5→10	1	1		1	D					$\overline{D}$		
Boolean difference												
Test	1	1	1	1	D	0	0	1	$\overline{D}$	$\overline{D}$	1	D

D-algorithm

- **5 valued logics**
  - **Singular covers**
  - **Fault injection**
  - **Error propagation**

Intelligent search algorithms

- **Free parameters in a systematic search algorithm used for optimization**
  - Decisions
    - Which fault?
    - Which algorithm?
    - Which direction etc.
  - Global criteria
    - At first the most difficult or most easy subproblem
    - Testability measure as guiding cost heuristics in the search

Goal of the testability analysis	Design for testability	Costs of the testing
	ATPG acceleration	Optimal decision making
	Random ATPG	Test length and/or fault coverage
Testability measure	Controllability	Setting a node to “0” or “1”
	Observability	Reading the value at a node
Circuit	Combinatorial	Sensitization to/from the fault location
	Sequential	Minimal number of clock phases
Level	Circuit	For all faults statically
	Fault	Each fault statically
	Value assignment	Dynamic
Factors	Topology	Distance
	Function	Difficulty

Fault simulation

- **Phases:**
  - **Specification:**
    - **Fault-free network N**
    - **Set of faulty networks after fault collapsing: {N_f}**
  - **Injection of the fault(s) into the network**
  - **Estimation and propagation of the errors**
  - **Failure detection**
  - **Removing the detected fault(s) from the list of faults**
- **Serial fault simulation: for *i=1* to *number_of_faults* do simulate (*faulty_network*),**

Simultaneous simulation of multiple faults in a single pass

- **Parallel**
  - **Word length w of the computer > 1, logic operations simultaneously on all bits,** $\left\lceil \frac{F}{w} \right\rceil$ **passes**

N_{w-1}			N_i	...	N_1	N

  - **Elements described by logic equations, fault modelling:**

$$v'_i = v_i \cdot \overline{m_{s-a-0,\,i}} + m_{s-a-1,\,i} \qquad \text{or} \qquad v'_i = v_i \cdot \overline{faulty_i} + Faultvalue_i$$

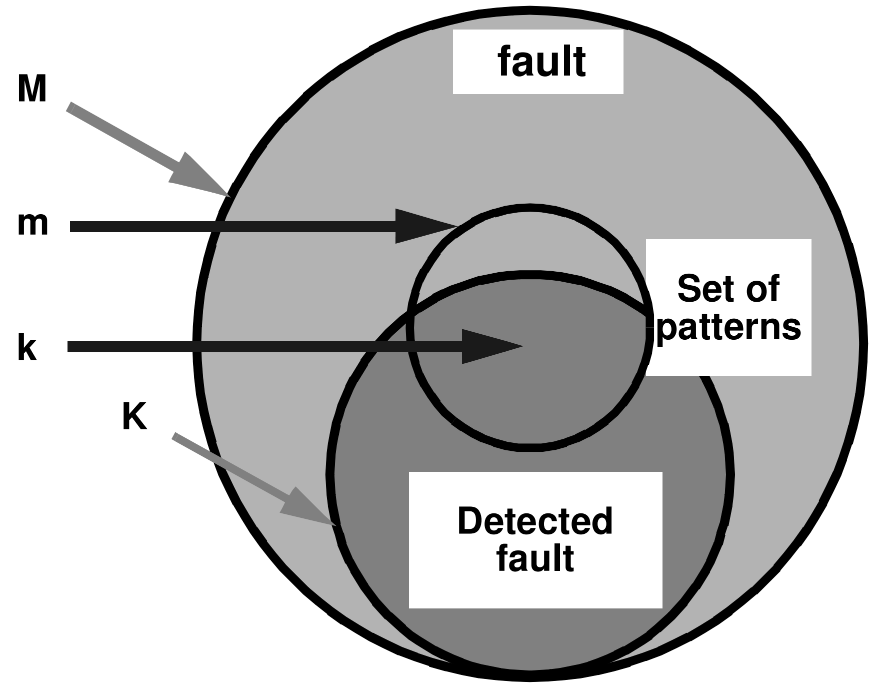
  - **Simultaneous simulation of independent faults: effect on disjunct set of nodes limited**
    - simultaneous simulation in identical bit positions
  - **Minimal upper bound: number p of the outputs, minimal number of passes:** $\left\lceil \frac{F}{wp} \right\rceil$
- **Deductive: simultaneous estimation of the node values in all faulty networks**
  - **Fault-free network simulated**
  - **Idea: the differences form a sparse vector ⇒ representation in a list form**
  - **List events: no changes in the value, but recalculation of the list**
- **Concurrent: only "visible" faults in {N_f} differing from N in behavior simulated**
- **Comparison (1<2<3)**

Method	Logic	Description of the function	Delay model	Run time	Storage
Parallel	±2	-	limited	3	1

Method	Logic	Description of the function	Delay model	Run time	Storage
Deductive	±2	-	limited	2	2
Concurrent	arbitrary	feasible	feasible	1	3

Stochastic fault simulation

- Goal: estimation of the fault coverage based on a sampling based fault simulation experiments



- M number of faults in the complete model
- K number of detected faults
- F fault coverage
- m cardinality of the pattern set
- k number of the detected faults in the sample

- $\hat{T}$ **estimator of the fault coverage**

Hypergeometric distribution

$$P(\ ) = P(\ ) = \frac{(\ )\times(\ )}{(\ )}$$

- **Approximation with normal distribution:**
  - **Mean value** $\mu_F = \quad = \frac{K}{M} = F$
  - **Standard deviation** $\sigma_F = \sqrt{\quad} = \sqrt{\frac{K}{M} \times (\quad) \times \frac{M-m}{M-1} \times \frac{1}{m}}$
  - **If** $M \gg m \gg 1$ **,** $\sigma_F = \sqrt{\frac{K}{M} \times \left(1 - \frac{K}{M}\right) \times \frac{M-m}{M-1} \times \frac{1}{m}} \approx \sqrt{\frac{K}{M} \times \left(1 - \frac{K}{M}\right) \times \frac{1}{m}} = \frac{\sqrt{F \times (1-F)}}{\sqrt{m}}$
  - **By a confidence level of 99.7%,** F **lies in** $[F - \sigma_F, F + \sigma_F]$ **, i.e.** $e_{max} = \frac{3\sqrt{F \times (1-F)}}{\sqrt{m}}$
  - **Maximal fault** $e_{max} = \frac{3\sqrt{F \times (1-F)}}{\sqrt{m}}$

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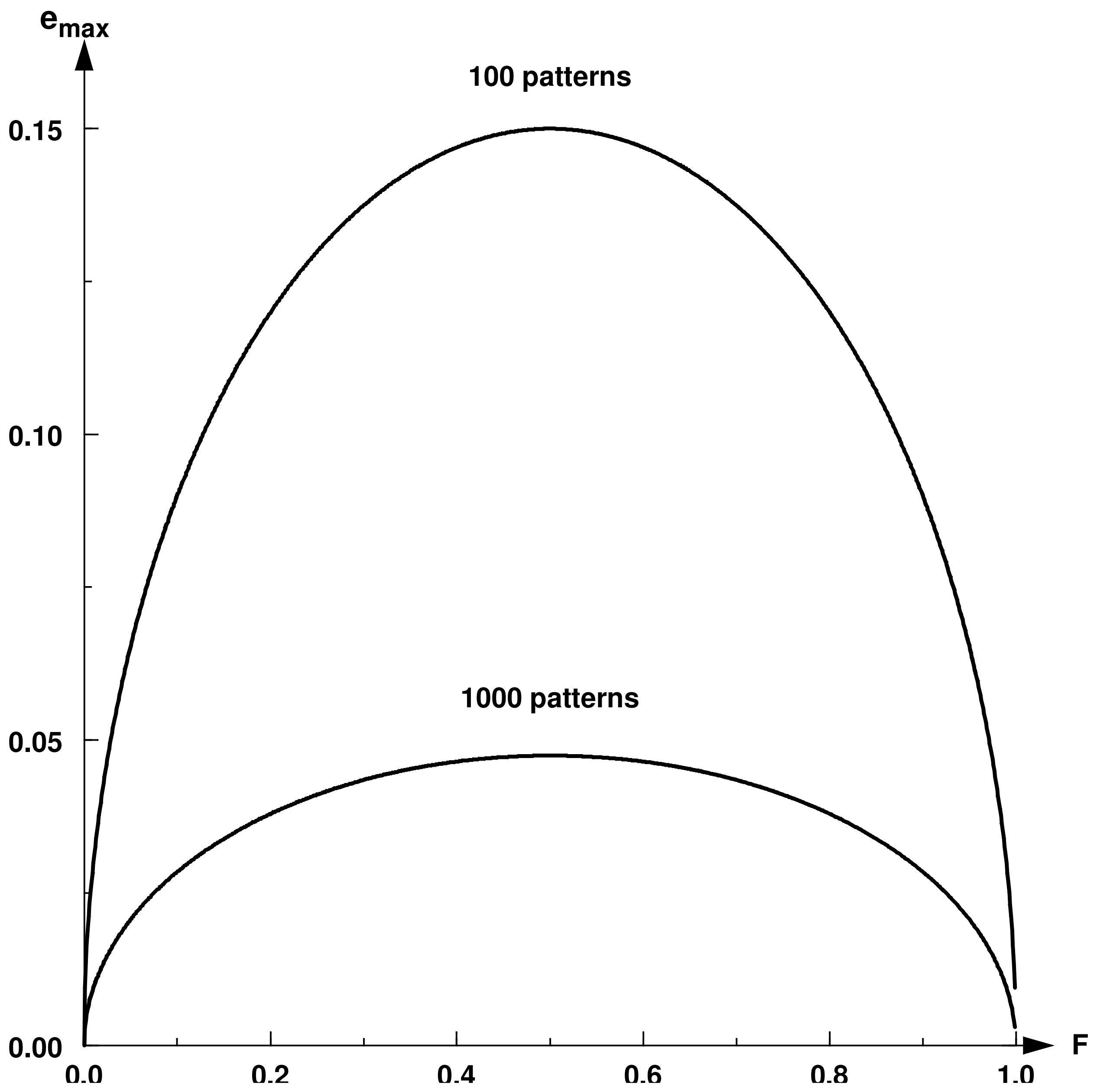
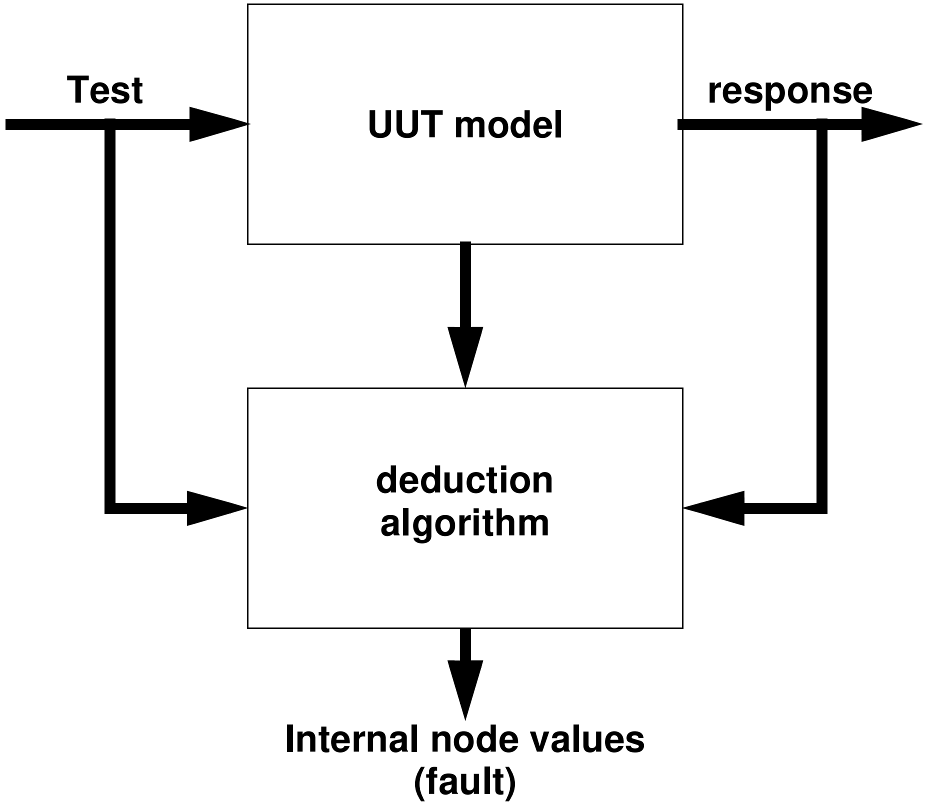
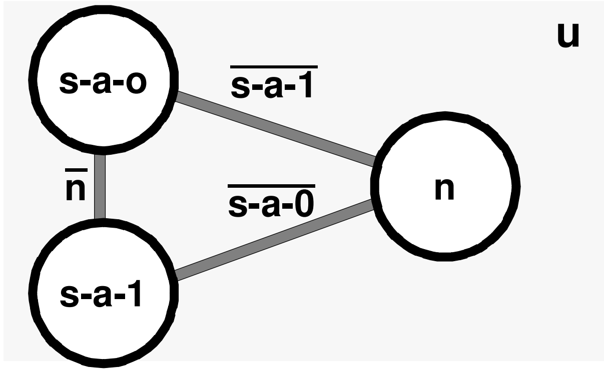

e max
100 patterns
1000 patterns
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
F

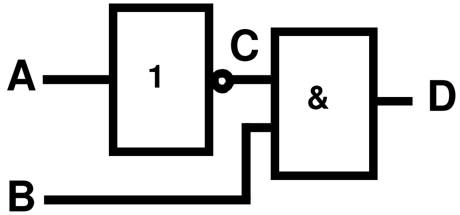
Effect-cause analysis



- **State space of a node:**
    - **normal**
    - **s-a-0**
    - **s-a-1**
    - **normal**
    - **s-a-0**
    - **s-a-1**
    - **unknown**



- **Reconstruction *of all fault situations* compatible with the test response without explicit fault enumeration (permanent single or multiple stuck-at faults)**
- **Example:**



Test	Reference		Actual value
	A	B	D
t_1	0	1	1
t_2	1	1	0
t_3	0	0	1

Deduction scheme

Effect	Consequence	Fault state				Reference			
		A	B	C	D	A	B	C	D
Initialization		u	u	u	u	0/x 1/x 0/x	1/x 1/x 0/x	1/x 0/x 1/x	1/1 0/0 0/1
$t_1(D)=1$	D: $\overline{\text{s-a-0}}$	u	u	u	$\overline{\text{s-a-0}}$	0/x 1/x 0/x	1/x 1/x 0/x	1/x 0/x 1/x	1/1 0/0 0/1
$t_2(D)=0$	D: $\overline{\text{s-a-1}} \Rightarrow$ D: $\{\overline{\text{s-a-0}}\} \cap \{\overline{\text{s-a-1}}\} \Rightarrow$ D: n	u	u	u	n	0/x 1/x 0/x	1/x 1/x 0/x	1/x 0/x 1/x	1/1 0/0 0/1
D: n	$t_1(B)=t_1(C)=1$ $t_3(B)=t_3(C)=1$	u	u	u	n	0/x 1/x 0/x	1/1 1/x 0/1	1/1 0/x 1/1	1/1 0/0 0/1
$t_1(C)=1$	C: $\overline{\text{s-a-0}}$	u	u	$\overline{\text{s-a-0}}$	n	0/x 1/x 0/x	1/1 1/x 0/1	1/1 0/x 1/1	1/1 0/0 0/1
$t_3(B)=1$ Reference: 0	B: s-a-1	u	s-a-1	$\overline{\text{s-a-0}}$	n	0/x 1/x 0/x	1/1 1/x 0/1	1/1 0/x 1/1	1/1 0/0 0/1

Effect	Consequence	Fault state				Reference			
		A	B	C	D	A	B	C	D
B: s-a-1	t_2(B)=1	u	s-a-1	$\overline{\text{s-a-0}}$	n	0/x 1/x 0/x	1/1 1/1 0/1	1/1 0/x 1/1	1/1 0/0 0/1
t_1-t_2: D: 1$\rightarrow$0 B: const.	A: n C: n	n	s-a-1	n	n	0/0 0/1 0/0	1/1 1/1 0/1	1/1 0/0 1/1	1/1 0/0 0/1

- **Operations:**
  - **Horizontal implication (Signal path)**
  - **Vertical (time related) implication**

Forced value:

- **Forced value at node K during test t_i: $FV(K_i)=w$**
- **Node K is *forced to w*, if in all fault situation during t_i: $K_i=w$ OR during all test: $K \equiv \overline{w}$**
- **All inputs and all nodes driven by normal nodes are forced**
- **All inputs of a gate forced identically during a test t $\Rightarrow$ the output forced as well**

Example: gate Y=AND (A,B)

Forced value		Reference	Actual value		Fault state at Y		
A	B	Y	A	B	n	s-a-1	s-a-0
$A^t=1$	$B^t=1$	$Y^t=1$	$A^t=1$	$B^t=1$	$Y^t=1$	$Y\equiv1$	$Y\equiv0$
			$A\equiv0$	$B=x$	$Y\equiv0$		
			$A=x$	$B\equiv0$			
$A^t=0$	$B^t=0$	$Y^t=0$	$A^t=0$	$B=x$	$Y^t=0$	$Y\equiv1$	$Y\equiv0$
			$A=x$	$B^t=0$			
			$A\equiv0$	$B=x$	$Y\equiv0$		
			$A=x$	$B\equiv0$			
			$A\equiv1$	$B\equiv1$	$Y\equiv1$		
$A^t=1$	$B^t=0$	$Y^t=0$	$A^t=1$	$B^t=0$	$Y^t=0$	$Y\equiv1$	$Y^t=0$
			$A\equiv0$	$B=x$	$Y\equiv0$		
			[redacted]				
			$A\equiv1$	$B\equiv1$	$Y\equiv1$		

- **Vertical (time related) implication:**
  - $\exists i: K_i \neq FV(K_i) \Rightarrow K \equiv \overline{FV(K_i)}$ or $\exists i: K_i = FV(K_i) = w \Rightarrow \forall j: FV(K_j) = w: K_j = w$